Toppling Mechanism: resolving the question of alignment of slope and discontinuities

P.M. Maurenbrecher
University of Technology, Delft, The Netherlands
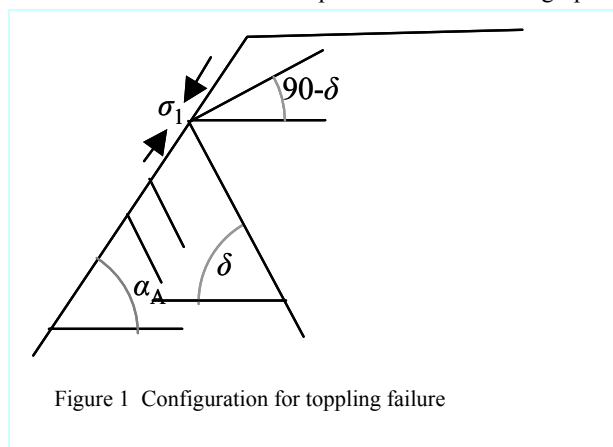
Dr. H.R.G.K. Hack
ITC International Institute for Geo-Information Science and Earth Observation, Enschede, The Netherlands

ABSTRACT:. The kinematic test for toppling developed by Goodman and Bray (1976) was based on a two dimensional relationship $(90°-\delta) + \varphi_j < \alpha$ (where δ= dip of discontinuity, φ_j, = friction angle of the discontinuity and, α = slope angle). The steeply dipping discontinuities into a slope, which the relationship describes, must have a strike nearly parallel to the strike of the slope. 'Nearly' is defined as up to 30° out of alignment though in the original paper by Goodman and Bray the strike had to be within 10°. By using the expressions for toppling failure and substituting instead for δ the apparent dip δ_a in Hack (1996) it is shown that the zone for toppling can be much larger than the 30° as stated by Goodman (1989). The relationship is derived in terms of principal stresses and apparent dip. The equation is more akin to a phenomena describing limited flexural slip at the surface of the slope resulting in 'spalling' of rock fragments from the slope surface. By making a distinction between toppling resulting from 'over-turning' and toppling by 'spalling' results in the much more limited instability zone on streographic projections as described by Goodman 1989. It is the the spalling that would eventually undermine the layering to form 'leaning columns' which can topple by 'over-turning' as the term 'topple' suggests. The slenderness of the columns and the dip of the layering ('lean of the columns') would indicate the likelihood of over-turning.
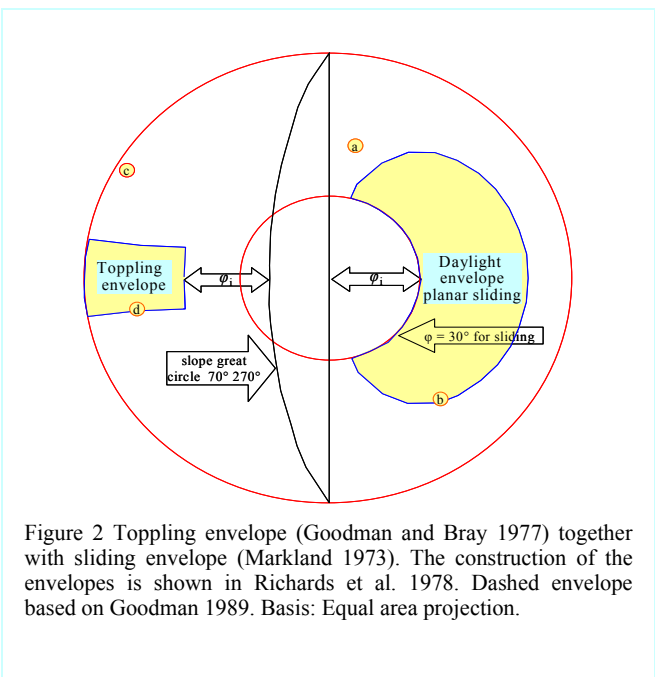
1 INTRODUCTION

Toppling failure mechanism in rock slopes (and also observed in stiff clay slopes) is less obvious than sliding failures such as planar or wedges. The model used by the original paper on this subject from Goodman and Bray (1967) shows a discontinuity dipping downwards into a slope, which kinematically allows for no movement (Fig. 1). This is evaluated further by Cruden (1989). Yet slopes observed in the field with this configuration show distress, usually in the form of flaking/spalling at the surface resulting in gradual deterioration of the slope to a less steep angle and development of scree deposits at its base. Sufficient undermining of the layers could result in rock blocks 'toppling' from the rock face. The equation $(90°-\delta) + \varphi_j < \alpha$ is the condition for flexural interlayer-slip which precedes toppling by 'over-turning'.

The condition has been plotted on a stereographic projection to show which poles of δ plot within an envelope for toppling. The envelope shows that for discontinuities having apparent dips δ_a steeper than $(90°-\alpha) + \varphi_j$ (re-writing the toppling equation) toppling can occur as long as the true dip direction does not exceed a direction than 10° the opposite direction of the slope angle. In Goodman 1988 this has been increased to 30°, stating that 15° (!) has been found too small. The 30° limit probably defines the condition for full toppling by overturning from the initial spalling mechanism. For poles of discontinuities exceeding

Figure 1 Configuration for toppling failure

Figure 2 Toppling envelope (Goodman and Bray 1977) together with sliding envelope (Markland 1973). The construction of the envelopes is shown in Richards et al. 1978. Dashed envelope based on Goodman 1989. Basis: Equal area projection.

this value full toppling is less likely to develop so that the only instability remains degradation of the slope through a process of spalling only. Figure 2 shows the kinematic condition as a stereographic projection. The model persists and was introduced for the first time into 'Rock Slope Engineering' 4th Edition by Wyllie and Mah (2004) from Hoek and Bray (1981) 3rd edition using a direction up to 10° out of alignment between the true dip of the discontinuity and opposite direction of the slope angle direction. Hence the need after thirty years to resolve the question for the toppling mechanism and the alignment condition of the discontinuity dip directions with respect to the slope angle direction.

2 FORMULATION TOPPLING RELATIONSHIP IN 3-D

To resolve the question of strike alignment of the discontinuity and that of slope one could instead examine the slope steepness in the opposite direction of the discontinuity dip. In the extreme case if the discontinuity dip direction is the same as the strike of the slope, the slope steepness is zero. A slope having a fall line steepness of 70° has a steepness varying from 70° at the fall line direction and 0° at the strike direction. A discontinuity of dip 80° can induce toppling failure for a slope of minimum steepness of $\alpha=10°+\varphi$. With a value of φ as low as 22°, for example shear surfaces in faulted peridotites in Oman (Maurenbrecher et al. 1990) a slope of steepness 32° will be at limiting equilibrium. For a 70° slope the 32° steepness occurs at ± 75° from the direction of 70° slope fall line. Even a φ of 55°would be at limiting equilibrium for a slope steepness of 65° which for a 70° slope occurs at ± 38° from the fall line. This exceeds the ±30° limit recommended by Goodman (1989).

Hack (1996) recommends calculating the apparent dip of the discontinuity in the direction of the slope fall line. Either way the same result is produced using the apparent slope steepness or apparent discontinuity dip resulting in the following relationship:
Toppling if

$-90°-\delta_a + \alpha>0°$ and $\delta_a<0°$

(Apparent dip, Hack loc cit., is negative for a direction oppositre to that of the slope fall line).
If this criteria is satisfied then the discontinuity properties have to be examined, essentially roughness, waviness, aperture and infill. These values are weighted from which a more likely friction value of the discontinuity is obtained.

3 MOHR-COULOMB RELATIONSHIP GOODMAN-BRAY CRITERION

Goodman and Bray (1977) criterion can be derived using the Mohr-Coulomb analysis with the principal stress σ_1 parallel to the slope. The other principal stresses σ_2 is zero (strictly atmospheric pressure acting on the slope surface) and σ_3 acts in a direction of the strike line of the slope and is neglected.

The analysis only applies to the discontinuity at the slope face and hence does not apply to conditions within the rock mass where the stress conditions change so that not only σ_2

increases in value but that σ_1 and σ_2 rotate. As stated in the introduction the kinematics of the Goodman-Bray criterion do not allow for any movement. One may have to assume that at macroscopic level differential flexural slip occurs due to shear and that this sets a train of events in motion causing gradual weakening and removal of rock fragments to such an extent that part of the layer blocks are undermined causing them to 'topple' from the rock face by over-turning.

The Mohr-Coulomb solution is presented in would be as follows (based on the model in Figure 1.):

Stress component initiating slip: $\sigma_1.\sin(\alpha-(90°-\delta))$

Stress component resisting slip: $\sigma_1.\cos(\alpha-(90°-\delta)).\tan\varphi$

Slip occurs if $\sigma_1.\sin(\alpha-(90°-\delta))> \sigma_1.\cos(\alpha-(90°-\delta)).\tan\varphi$

$\sin(\alpha-(90°-\delta))/.\cos(\alpha-(90°-\delta))>\tan\varphi$

$\tan(\alpha-(90°-\delta))>\tan\varphi$, $\alpha-(90°-\delta)>\varphi$, or $\alpha>(90°-\delta)+\varphi$

In the stereographic projection $\alpha-\varphi >(90°-\delta)$ is used which states if the angle of the slope less the friction angle is greater than the dip of the pole of the discontinuity (flexural) slip will occur.

The Mohr-Coulomb form of the Goodman-Bray criterion also allows for determination of a factor of safety:

$F= \tan\varphi/\tan (\alpha-(90°-\delta))$.

This equation allows examination of the influence of vertical discontinuities and steeply dipping discontinuities in the direction of the slope. Not surprisingly the factor of safety reduces further to zero to the point where the discontinuities are parallel to the slope. The Mohr-Coulomb model shows that the principal stress σ_1 component initiating slip far exceeds the normal component mobilizing shear resistance. Further rotation of the discontinuities would result in a sliding mode with a flexural-buckling mode in between.

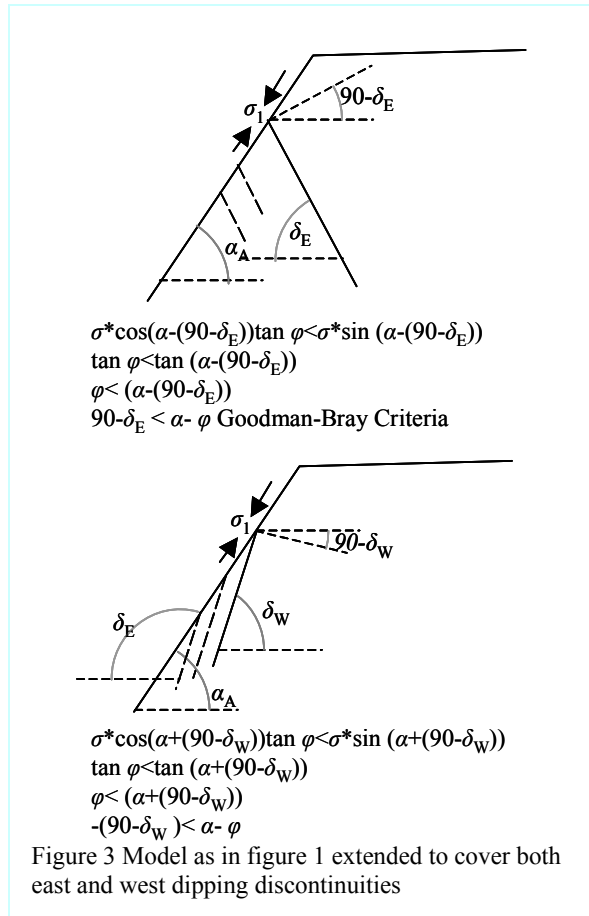## 4 STEREOGRAPHIC TOPPLING ENVELOPE

4.1 *Flexural Slip (spalling phenomena)*

Figure 2 shows the a 'lower hemisphere' stereographic projection with both a toppling failure envelope as published by Goodman and Bray (1977) and Goodman (1989) and the planar sliding envelope originally developed by Markham (1973). The method of a step-by-step construction is given by Richards et al. (1978) and again in Richards and Atherton, 1987. A much extended 'toppling' envelope would be required to allow for the Mohr-Coulomb model, both for apparent dips (or slopes) and discontinuities dipping more steeply than the fall line of the slope but in the general direction of the fall line.

The original 'Goodman-Bray' criterion states

$90-\delta_E <\alpha_W - \varphi$.

The stereographic envelope states, besides the above,

$\sigma*\cos(\alpha-(90-\delta_E))\tan\varphi<\sigma*\sin(\alpha-(90-\delta_E))$
$\tan\varphi<\tan(\alpha-(90-\delta_E))$
$\varphi<(\alpha-(90-\delta_E))$
$90-\delta_E<\alpha-\varphi$ Goodman-Bray Criteria

$\sigma*\cos(\alpha+(90-\delta_W))\tan\varphi<\sigma*\sin(\alpha+(90-\delta_W))$
$\tan\varphi<\tan(\alpha+(90-\delta_W))$
$\varphi<(\alpha+(90-\delta_W))$
$-(90-\delta_W)<\alpha-\varphi$

Figure 3 Model as in figure 1 extended to cover both east and west dipping discontinuities

$0<\delta_E<90$ and $0<\varphi<\alpha_W$

for the discontinuity strike coinciding with that of the slope (for illustrative purposes the strikes are considered North-South and slope facing west, bearing 270°). Figure 3 shows the models used for analyzing the failure mode both for δ_E and δ_W.

$\delta_W\equiv180>\delta_E>90$ so that the first limit

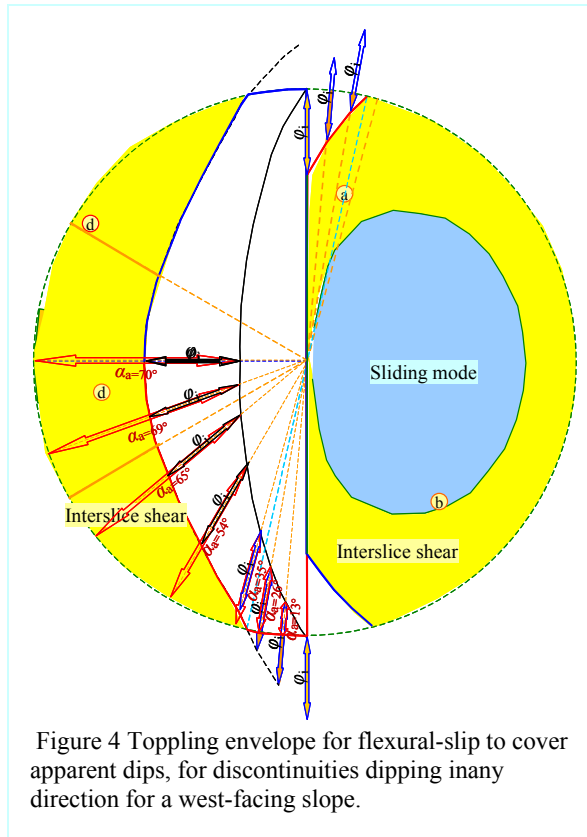$0<\delta_E<90$ becomes $0<\delta_E<180$.
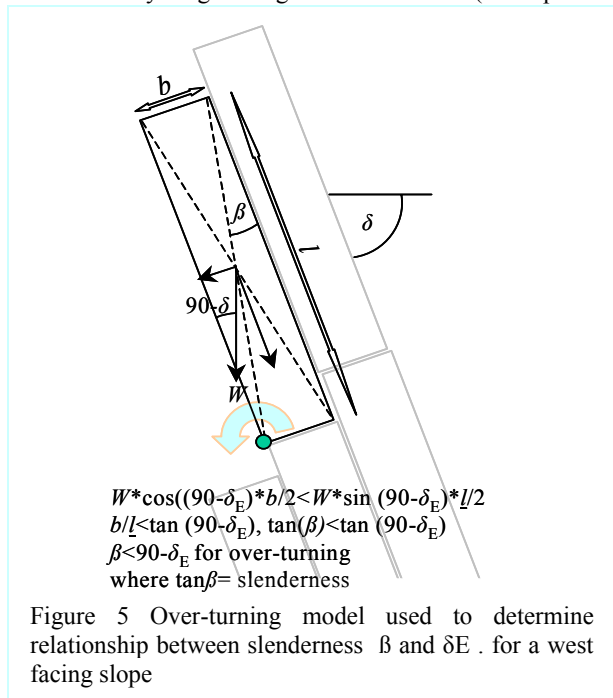
The second limit for friction would be

$0<\varphi<90°$.

Goodman (1989) also sets the strike alignment, denoted as ω, of the discontinuity to that of the slope at a maximum of 30°. (Note: Though insignificant, all publications show an incorrect plot for values of ω for either 10° in Goodman and Bray (1977), Richards et al. (1978, 1987) and Wyllie (2004) and 30° in Goodman (1989). The method of plotting is given in Richards et al. (1979, 1987)).

The Mohr-Coulomb failure criterion allows flexural slip to occur up to values of ω up to 90° by substituting the apparent slope angle or the apparent dip of the discontinuity. The resultant envelope presented for two values of φ is given in figure 4.

Figure 4 Toppling envelope for flexural-slip to cover apparent dips, for discontinuities dipping in any direction for a west-facing slope.

4.2 Overturning Condition

Hack (1996) recommends that the envelope be restricted to $\delta_E<85°$. Owing to constant spalling of the rock eventually the slope steepens resulting in overhanging columns that can overturn. Considering a column (Figure 5) resulting from inter-layering of length l and breadth b (corresponding
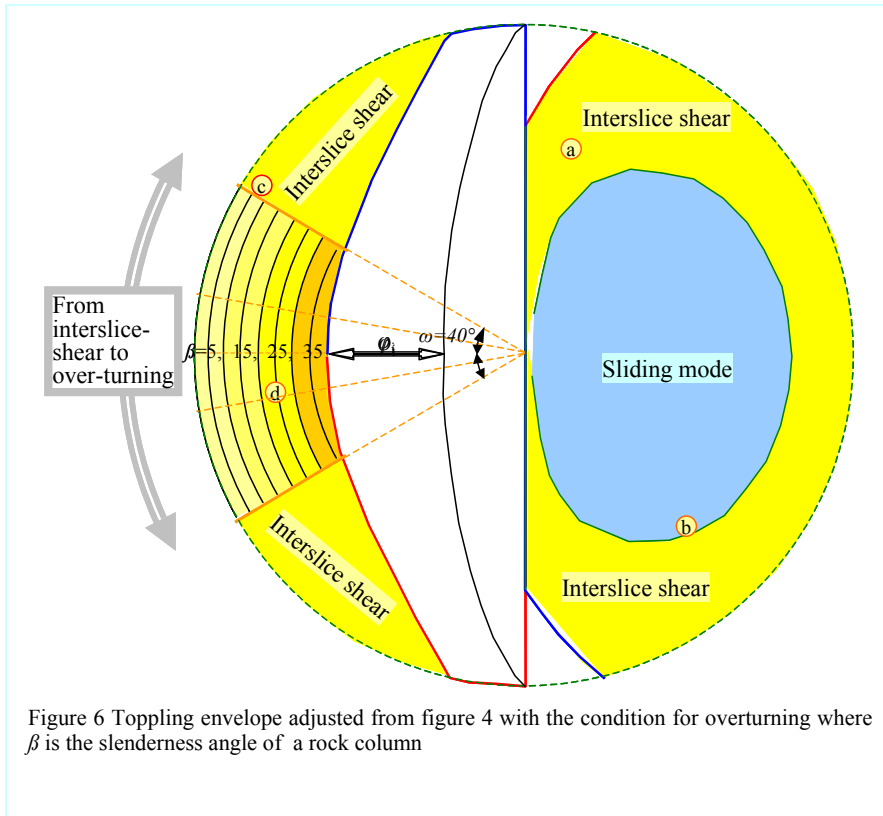

$W*\cos((90-\delta_E)*b/2<W*\sin(90-\delta_E)*l/2$
$b/l<\tan(90-\delta_E)$, $\tan(\beta)<\tan(90-\delta_E)$
$\beta<90-\delta_E$ for over-turning
where $\tan\beta=$ slenderness

Figure 5 Over-turning model used to determine relationship between slenderness β and δE. for a west facing slope

Figure 6 Toppling envelope adjusted from figure 4 with the condition for overturning where β is the slenderness angle of a rock column

slip' restricted to the discontinuities daylighting at and near to the surface of the slope. This form of instability occurs within a much larger envelope than that shown in existing publications and would result in spalling of rock fragments from the surface of the rock slope. The spalling of the rock can be described as surface degradation due to overstressing in shear. The degradation of the slope eventually, with time, results in selective undermining of layers. For sufficiently slender steeply dipping layers toppling by over-turning may then result. This depends on the slenderness ratio of the resulting 'columns' where dips of the layers steeper than 85° probably will not topple or where the alignment of the strike of the layering exceeds 30° the alignment of the strike of the slope. These two conditions however would not preclude the spalling phenomena based on the inter-layer flexural slip model used originally to define the 'toppling' failure mechanism.

to layer spacing) then the ratio can be expressed as tan β = b/l.

When $\beta < 90 - \delta_E$ (by taking moments about the overturning point of the column block, see Figure 5) the column will overturn. Hence the overturning condition can be added to the toppling stability envelope. For example when δ_E approaches 85° dip tan β =tan 5° ≈ 0.1. Hence a spacing of b=0.5m would require intact columns of l= 5m. As β is reduced to 2° the columns would require a height of 14m (equivalent to a 4-story building). Such situations are seldom observed in slopes with the possible exception of special columnar basalts.

The other restriction for toppling failure is the alignment of the strike of the slope and that of the discontinuity. When the alignment angle value increases so would the value b increase making over-turning toppling less likely unless a tertiary joint system (in addition to the 'tension' joint developing at the base of the column) exists to allow dislodgement before the ratio b/l is too large to satisfy the overturning condition.

In Figure 6 stereographic envelopes are presented showing the influence of over-turning based on the slenderness values of β and on the discussion with regard to alignment of the strike of the slope with that of the discontinuities superimposed on the envelopes for flexural-slip toppling.

5 CONCLUSIONS

The term 'toppling' for describing the condition for discontinuities dipping into the rock mass should be qualified as the model used to produce this type of instability would be better described as 'flexural interlayer

REFERENCES

Cruden D.M. (1989) Limits to common toppling. Canadian Geotechnical Journal; 1989 Vol. 26, no 4. pp. 737-742.

Goodman, R.E. & Bray, J.W. 1976, Toppling of rock slopes In: Rock engineering for foundations and slopes; proceedings of a specialty conference, Vol. 2 p201-233, Am. Soc. Civ. Eng.. New York,

Hack, H.R.G.K. 1996 Slope Stability Probability Classification: SSPC, PhD Thesis TU Delft, ITC Publication number 43, 258p

Markland, J.T. (1972) A useful technique for estimating the stability of rock slopes when rigid wedge type of failure is expected, Imperial College Rock Mechanics Research Report No. 19

Maurenbrecher, P.M., J. James & G. de Lange, 1990, "Major road-cut design in rock, Muscat Capital Area, Oman" International Conf. on Mechanics of Jointed Rock, 18/20 April 1990, Vienna Editor: Hans Peter Rossmanith, Balkema, Rotterdam, p929-935

Richards, L.R. and Atherton, D. 1987. Stability of rock slopes. Ground Engineer's reference book. Editor: Bell, F.G., Butterworths, London 12: 3-16

Richards, L.R., G.M.M. Leg, R.A. Whittle, 1978, Appraisal of stability conditions in rock slopes, in Foundation Engineering in Difficult Ground (Bell, F.G. editor) Butterworths p449-512

Wyllie, D.C. and C.W. Mah 2004 Rock Slope Engineering Civil and Mining, new 4th Edition Spon Press Taylor & Francis Group, Andover Hants UK